International Journal of Theoretical Physics, Vol. 45, No. 9, September 2006 (© 2006)
DOI: 10.1007/s10773-006-9091-6

Quantum Entanglement of Many Particles in Spinor Bose–Einstein Condensates

Lixin Xia,[1,2,3] and YiMing Xie[1]

Published Online: September 28, 2006

We present a theoretical treatment of the proposal for creating maximally entangled states of many particles in spin-1 Bose–Einstein condensates (BECs) by applying a single atom Raman transition [You. L. (2003). *Physical Review Letters* 90, 030402]. It is shown that the three-mode model suggested by You can be further reduced to an efficient two-mode one by a simple method. We also suggest a scheme for generating the atom-atom continuous-variable entangled states in this system.

KEY WORDS: quantum entanglement; spinor BEC; Raman transition.

PACS number: 03.75.Gg, 03.75.Mn, 05.30.JP, 03.75.Hh

1. INTRODUCTION

Recently, quantum entanglement, entanglement in short, has been used to perform many useful works in quantum information processing (Loyd, 1993; Bennett and Wiesner, 1992; Bennett *et al.*, 1993; Cleve *et al.*, 1999). The creation and manipulation of the many particles entangled states is of significant interest, since the whole field of the quantum computation and quantum information science is based on such a ability. Methods for creating entangled states have been found for various physical systems, including nonlinear optics, ion traps, cavity quantum electrodynamics, and nuclear magnetic resonance Nielsen and Chuang (2000). On the other hand, the study of the entanglement characteristics of various interacting many-body system has also given exciting new insight into fundamental aspects of quantum physics.

The experimental observation of Bose–Einstein condensates (BECs) opened a new prospect in the generation of the entangled states and the implementation of quantum information precessing. The feature of the long coherence time also makes proposals for engineering many particles entanglement feasible. There have

[1] Department of Physics, Tsinghua University, Beijing 100084, China.
[2] Department of Physics, Hunan Normal University, Changsha 410081, China.
[3] To whom correspondence should be address at Department of Physics, Hunan University of Science and Engineering,Yongzhou, 425006, China; e-mail: xialixin@hunnu.edu.cn.

been several proposals for controlling and engineering many particle entangled states by using a weakly interacting BEC with internal degree of free or confined in a double-well potential. For example, by appropriately controlling the interaction between atoms, the ground state can become an entangled state in a weakly coupled double-well potential Cirac *et al.* (1998); Steel and Collett (1998); Search *et al.* (2001). It was also shown that by its nature every BEC is in a highly entangled state Simon (2002); Hines *et al.* (2003). Sørensen *et al.* suggested creating massive entangled spin squeezed state from a two-component condensate using the inherent atom-atom interaction Sørensen *et al.* (2001). In Ruostekoski *et al.* (1998), the authors proposed a method for creating Schrödinger cat states in BECs by means of scattering light from two moving with oppositive velocities. Moreover, by controlling the dynamics, the system of the two-component BECs can evolve into a Schrödinger cat state starting from certain initial states Gordon and Savage (1999); Helmerson and You (2001); Micheli *et al.* (2003); You (2003); Gerry and Campos (2003). Latterly, spin-exchange interaction of a spinor condensate Stenger *et al.* (1998); Barrett *et al.* (2001) was also proposed as a candidate for generating pairs of atom. Comparing to the above-mentioned system, a spinor BEC has richer physics, although the two-component BECs can be viewed as quasi-spin-$1/2$. Coherent spin-exchange collisions are used to create an entangled state between atoms with hyperfine spin state $+1$ and -1 for an initial condensate in atomic hyperfine spin state 0 without the need of light fields Pu and Meystre (2000); Duan *et al.* (2000). The work of Duan *et al.* further showed that the three-mode entanglement can be generated in the spin-1 BECs by free dynamical evolution with properly initial states Duan *et al.* (2002). More recently, a protocol was suggested to create maximally entangled states in such a spin-1 BEC by driving a single atom Raman transition using the classical laser in the Mott insulator state You (2003). In this scheme, maximally entangled pair, triplets, and quartiles were considered between atoms with hyperfine states -1 and $+1$. But the work involves a single optical well and is limited to a small number of atom. In the terms of the idea of a quantum Zeno subspace Facchi and Pascazio (2002), the underlying mechanism for this protocol can be rather conveniently understood and can be extended to larger number of condensed atoms Zhang and You (2003). In fact, the authors retorted to the concept of the quantum Zeno subspace to reduce the three-mode Hamiltonian to an extensively studied two-mode one, which is known to generate maximally entangled state. Subsequently, Zou *et al.* presented further analysis of the model and showed that in strong coupling limit the model can be greatly simplified Pahlke and Mathis (2004). Pahlke and Mathis (2004) A scheme for how to generate the entangled state between hyperfine spin 0 and $+1$ is also given . In this paper, it is shown that the maximally entangled states of many particles in spin-1 can be created by using a possibly simpler method You (2003); Kuang and Ouyang (2000). Our main attention is paid in investigating how to generate the atom-atom continuous-variable entangled states starting from coherent states in this system. Recently, such a continuous-variable entangled states have assumed a key role in continuous variable quantum information processing Kuang and Lan (2003).

2. MODEL AND SOLUTION

We consider a spin-1 condensate with N atoms, in which the spin states $|F=1, M_F=-1\rangle$ and $|F=1, M_F=+1\rangle$ are coupled by a classical laser pulse. The corresponding Hamiltonian takes the form $H = H_0 + H_{\text{int}}$, where H_0 and H_{int} describe the condensates and the coupling between the external field and the condensates, respectively. These terms for the condensate and the coupling are given in the second quantized form You (2003); Pahlke and Mathis (2004); Law *et al.* (1998)

$$
\begin{aligned}
H_0 = {} & \sum_{\alpha} \int d^3x \hat{\Psi}_{\alpha}^{\dagger} \left(-\frac{\nabla^2}{2M} + V_T \right) \hat{\Psi}_{\alpha} + \frac{\lambda_0}{2} \sum_{\alpha,\beta} \int \hat{\Psi}_{\alpha}^{\dagger} \hat{\Psi}_{\beta}^{\dagger} \hat{\Psi}_{\alpha} \hat{\Psi}_{\beta} d^3x \\
& + \frac{\lambda_2}{2} \int \big(\hat{\Psi}_{+1}^{\dagger} \hat{\Psi}_{+1}^{\dagger} \hat{\Psi}_{+1} \hat{\Psi}_{+1} + \hat{\Psi}_{-1}^{\dagger} \hat{\Psi}_{-1}^{\dagger} \hat{\Psi}_{-1} \hat{\Psi}_{-1} \\
& + 2 \hat{\Psi}_{+1}^{\dagger} \hat{\Psi}_{0}^{\dagger} \hat{\Psi}_{+1} \hat{\Psi}_{0} + 2 \hat{\Psi}_{-1}^{\dagger} \hat{\Psi}_{0}^{\dagger} \hat{\Psi}_{-1} \hat{\Psi}_{0} \\
& - 2 \hat{\Psi}_{+1}^{\dagger} \hat{\Psi}_{-1}^{\dagger} \hat{\Psi}_{+1} \hat{\Psi}_{-1} + 2 \hat{\Psi}_{0}^{\dagger} \hat{\Psi}_{0}^{\dagger} \hat{\Psi}_{+1} \hat{\Psi}_{-1} + 2 \hat{\Psi}_{+1}^{\dagger} \hat{\Psi}_{+1}^{\dagger} \hat{\Psi}_{0} \hat{\Psi}_{0} \big) d^3x, \quad (1)
\end{aligned}
$$

$$
H_{\text{int}} = \Omega \int d^3x (\hat{\Psi}_{+1}^{\dagger} \hat{\Psi}_{-1}^{\dagger} + \hat{\Psi}_{-1}^{\dagger} \hat{\Psi}_{+1}^{\dagger}) \quad (2)
$$

where M is the mass of the atom, $\hat{\Psi}_{\alpha}(\vec{x})$ $(\alpha = 0, \pm 1)$ denotes the annihilation operator for the $M_F = \alpha$ component of a spin-1 field. The trapping potential V_T is assumed to be spin-independent. The interaction parameters are $\lambda_0 = 4\pi\hbar^2(a_0 + 2a_2)/(3M)$ and $\lambda_2 = 4\pi\hbar^2(a_2 - a_0)/(3M)$ with a_f $(f = 0, 2)$ being the s-wave scattering length for spin-1 atoms in the combined symmetric channel of total spin f. For the two experimentally realized spinor condensate systems (^{23}Na and ^{87}Rb), we have $|\lambda_2| \ll \lambda_0$. Ω is the coupling strength between the fields and condensates. Under the single mode approximation in which atoms in different spin state are described by the same wave function $\phi(\vec{x})$ Yi et al. (2002), we can expand the atomic field operator: $\hat{\Psi}_{\alpha} \approx \hat{a}_{\alpha} \phi(\vec{x})$, such that the system can be reduced to a three mode Hamiltonian

$$
\begin{aligned}
H = {} & -\lambda_2' \big(\hat{a}_0^{\dagger 2} - 2 \hat{a}_{+1}^{\dagger} \hat{a}_{-1}^{\dagger} \big) \big(\hat{a}_0^2 - 2 \hat{a}_{+1} \hat{a}_{-1} \big) \\
& + \Omega \big(\hat{a}_{+1}^{\dagger} \hat{a}_{-1} + \hat{a}_{-1}^{\dagger} \hat{a}_{+1} \big) \quad (3)
\end{aligned}
$$

In deriving the above Hamiltonian, we have neglected some terms related to the total number operator $\hat{N} = \hat{a}_{+1}^{\dagger} \hat{a}_{+1} + a_0^{\dagger} \hat{a}_0 + a_{-1}^{\dagger} \hat{a}_{-1}$, which is a constant operator.

Here $2\lambda_0' \equiv \lambda_0 \int |\phi(\vec{x})|^4 \, d^3x$ and $\hat{a}_\alpha$ is the annihilation operator associated with the condensate mode satisfying the usual commutation relation $\left[\hat{a}_\kappa, \hat{a}_\gamma\right] = 0$ and $\left[\hat{a}_\kappa, \hat{a}_\gamma^\dagger\right] = \delta_{\kappa\gamma}$. In case of no coupling, with the help of the method developed in quantum optics, Law *et al*. have demonstrated that the ground state of the system may be a entangled state depending on the interaction between atoms Law *et al*. (1998). We note that the combined system cannot be solved analytically because of the presence of the term H_{int}. In order to get insight into the dynamics of such a three-component BEC system, some approximation is necessary. For a weak nonlinear interaction or strong coupling, a closed analytical solution can be obtained under the rotating wave approximation (RWA) Pahlke and Mathis (2004); Kuang and Ouyang (2000).

To begin the analysis, we introduce a new pair of bosonic operators $\hat{A}_{-1}$ and $\hat{A}_{+1}$ defined by You (2003); Kuang and Ouyang (2000)

$$\hat{a}_{-1} = \frac{1}{\sqrt{2}}(\hat{A}_{-1}\exp(i\Omega t) + \hat{A}_{+1}\exp(-i\Omega t)) \tag{4}$$

$$\hat{a}_{+1} = \frac{1}{\sqrt{2}}(\hat{A}_{-1}\exp(i\Omega t) - \hat{A}_{+1}\exp(-i\Omega t)), \tag{5}$$

which is similar to the dressed basis operator suggested by Zou *et al*. Here $\hat{A}_{-1}$ and $\hat{A}_{+1}$ are slowly varying operators, which satisfy the same bosonic commutation relation as $\hat{a}_{-1}$ and $\hat{a}_{+1}$. Then the Hamiltonian can be reexpressed as the form

$$\begin{aligned} H &= -\lambda_2'\left(\hat{a}_0^{\dagger 2}\hat{a}_0^2 + \hat{A}_{+1}^{\dagger 2}\hat{A}_{+1}^2 + \hat{A}_{-1}^{\dagger 2}\hat{A}_{-1}^2\right) \\ &\quad + \Omega\left(\hat{A}_{-1}^\dagger\hat{A}_{-1} - \hat{A}_{+1}^\dagger\hat{A}_{+1}\right) - H', \end{aligned} \tag{6}$$

$$\begin{aligned} H' &= \left(\hat{a}_0^{\dagger 2}\hat{A}_{-1}^2 + \hat{A}_{+1}^{\dagger 2}\hat{a}_0^2\right)\exp(i2\Omega t) \\ &\quad + \left(\hat{a}_0^{\dagger 2}\hat{A}_{+1}^2 + \hat{A}_{-1}^{\dagger 2}\hat{a}_0^2\right)\exp(-i2\Omega t) \\ &\quad - A_{+1}^{\dagger 2}A_{-1}^2\exp(i4\Omega t) - A_{-1}^{\dagger 2}A_{+1}^2\exp(-i4\Omega t) \end{aligned} \tag{7}$$

We also find that the total number operator $\hat{N} = \hat{A}_{+1}^\dagger\hat{A}_{+1} + a_0^\dagger\hat{a}_0 + A_{-1}^\dagger\hat{A}_{-1}$ is still a conserved constant. H' includes the terms oscillating with the frequency 2Ω and 4Ω. In the strong coupling limit, we can employ the RWA, where the quickly oscillating terms of the form $\exp(\pm i2\Omega t)$ and $\exp(\pm i4\Omega t)$ can be approximated by their zero average value. The work of ref. Pahlke and Mathis (2004) has pointed out that the approximation is a good one under the condition $\Omega \gg N\lambda_2'$. The RWA means $H' = 0$, so we obtain the approximation Hamiltonian

$$\begin{aligned} H &= -\lambda_2'\left(\hat{a}_0^{\dagger 2}\hat{a}_0^2 + \hat{A}_{+1}^{\dagger 2}\hat{A}_{+1}^2 + \hat{A}_{-1}^{\dagger 2}\hat{A}_{-1}^2\right) \\ &\quad + \Omega(\hat{A}_{-1}^\dagger\hat{A}_{-1} - \hat{A}_{+1}^\dagger\hat{A}_{+1}). \end{aligned} \tag{8}$$

It is also noticed in (8) that if the system is initially in the spin $+1$ and spin -1 subspace, the external field drives the system to remain the subspace. Since our main interest is the dynamics of such a subspace, we can obtain the resulting effective Hamiltonian

$$H_{\text{eff}} = \Omega(\hat{A}_{-1}^{\dagger}\hat{A}_{-1} - \hat{A}_{+1}^{\dagger}\hat{A}_{+1}) - \frac{\lambda_2'}{2}(\hat{A}_{-1}^{\dagger}\hat{A}_{-1} - \hat{A}_{+1}^{\dagger}\hat{A}_{+1})^2, \tag{9}$$

which is our starting Hamiltonian. In deriving (9), we also use the relation $\hat{N} = \hat{A}_{+1}^{\dagger}\hat{A}_{+1} + A_{-1}^{\dagger}\hat{A}_{-1}$ and drop the constant terms.

In order to demonstrate, how to create an entangled state between atoms, we introduce two Fock spaces of $(\hat{A}_{-1}, \hat{A}_{+1})$ and $(\hat{a}_{-1}, \hat{a}_{+1})$ in which the bases are defined by

$$|n)_{-1}|m)_{+1} = |n, m) = \frac{1}{\sqrt{n!m!}}\hat{A}_{-1}^{\dagger n}\hat{A}_{+1}^{\dagger m}|0, 0), \tag{10}$$

$$|n\rangle_{-1}|m\rangle_{+1} = |n, m\rangle = \frac{1}{\sqrt{n!m!}}\hat{a}_{-1}^{\dagger n}\hat{a}_{+1}^{\dagger m}|0, 0\rangle, \tag{11}$$

where n and m take nonnegative integers. Obviously, H_{eff} is diagonal in the Fock space of $(\hat{A}_{-1}, \hat{A}_{+1})$

$$H_{\text{eff}}|n, m) = E(n, m)|n, m), \tag{12}$$

$$E(n, m) = \Omega(n - m) - \lambda_2'(n - m)^2. \tag{13}$$

If we start with the BECs prepared in the Fock state $|0, N\rangle$, the state of the system at later time t is determined by

$$|\psi(t)\rangle = \exp(-iH_{\text{eff}}t)|0, N\rangle. \tag{14}$$

In Pahlke and Mathis (2004), such a initial state was expanded in terms of the eigenstates of operator of H_{int}. Here, note that, because $|0, 0\rangle = |0, 0)$, we can use (4) and (5) to obtain the useful connection between the two Fock spaces

$$\begin{aligned}
|0, N\rangle &= \frac{\hat{a}_{+1}^{\dagger n}}{\sqrt{N!}}|0, 0\rangle \\
&= \frac{1}{2^{N/2}\sqrt{N!}}(\hat{A}_{-1} - \hat{A}_{+1})^N|0, 0) \\
&= \frac{1}{2^{N/2}}\sum_{m=-N/2}^{m=N/2}\begin{pmatrix} N \\ N/2 + m \end{pmatrix}^{1/2}(-1)^{N/2-m}|N + m, N/2 - m),
\end{aligned} \tag{15}$$

such that the state at later time t can be reexpressed as

$$|\psi(t)\rangle = \frac{1}{2^{N/2}} \sum_{m=-N/2}^{m=N/2} \binom{N}{N/2+m}^{1/2} e^{i2\lambda_2' m^2 t} e^{-i2\Omega mt}$$
$$\times(-1)^{N/2-m}|N+m, N/2-m), \tag{16}$$

where $\binom{N}{N/2+m}$ is the binomial coefficient You (2003) (You, 2003). For simplicity, we assume that N is even, and choose the parameters Ω, λ' and a particular time t to satisfy $\Omega t = n\pi$ and $2\lambda_2' t = (2k+1)\pi/2$ with a resulting maximum entangled N-GHZ state You (2003) (You, 2003)

$$|\psi_N\rangle = \frac{1}{2^{(N+1)/2}\sqrt{N!}} \sum_{m=-N/2}^{m=N/2} \binom{N}{N/2+m} \hat{A}_{-1}^{N/2+m}$$
$$\times \hat{A}_{+1}^{N/2-m}(e^{-i\pi/4} + e^{i\pi/4}(-1)^{N/2-m})|0, 0)$$
$$= \frac{1}{\sqrt{2}} \left(e^{-i\pi/4} \frac{\hat{a}_{-1}^{\dagger N}}{\sqrt{N!}} + e^{i\pi/4} \frac{\hat{a}_{+1}^{\dagger N}}{\sqrt{N!}} \right) |0, 0\rangle$$
$$= \frac{1}{\sqrt{2}}(e^{-i\pi/4}|N, 0\rangle + e^{i\pi/4}|0, N\rangle) \tag{17}$$

Thus the analytical expression is given for the generation of the maximally entangled state between spin ±1. The similar procedure can be directly used to produce the maximally entangled state with initial state $|N, 0\rangle$. Comparing to the method suggested by Pahlke and Mathis (2004), this one may be more direct and simpler.

3. ATOM-ATOM CONTINUOUS-VARIABLE ENTANGLED STATES

In the above section, the initial states are assumed to be the Fock state or the number state. As a simple extension, we now investigate the case where the system is in the coherent states initially. Following Kuang and Lan (2003), we shall describe a method to engineer atom-atom continuous-variable entangled states. Consider two coherent states defined in Fock spaces of $(\hat{A}_{-1}, \hat{A}_{+1})$ and $(\hat{a}_{-1}, \hat{a}_{+1})$, respectively,

$$|\alpha_{-1}, \alpha_{+1}\rangle = D_{\hat{a}_{-1}}(\alpha_{-1}) D_{\hat{a}_{+1}}(\alpha_{+1})|0, 0\rangle, \tag{18}$$

$$|u_{-1}, u_{+1}) = D_{\hat{A}_{-1}}(u_{-1}) D_{\hat{A}_{+1}}(u_{+1})|0, 0), \tag{19}$$

where $D_{\hat{a}_i}(\alpha_i)$ and $D_{\hat{A}_i}(u_i)$ are the usual displacement operators defined by

$$D_{\hat{a}_i}(\alpha_i) = \exp(\alpha_i \hat{a}_i^\dagger - \alpha_i^* \hat{a}_i), \tag{20}$$

$$D_{\hat{A}_i}(u_i) = \exp(u_i \hat{A}_i^\dagger - u_i^* \hat{A}_i). \tag{21}$$

Due to $|0, 0\rangle = |0, 0)$, we can obtain a useful relation to connect $|\alpha_{-1}, \alpha_{+1}\rangle$ and $|u_{-1}, u_{+1})$ by a simple calculation

$$|\alpha_{-1}, \alpha_{+1}\rangle = |u_{-1}, u_{+1}), \tag{22}$$

$$|\alpha_{-1}, \alpha_{+1}) = |u_{-1}, u_{+1}\rangle, \tag{23}$$

where $u_{-1} = (\alpha_{-1} + \alpha_{+1})/\sqrt{2}$ and $u_{+1} = (\alpha_{-1} - \alpha_{+1})/\sqrt{2}$. Following the arguments of Bose broken symmetry, we assume that the two condensates are initially in the coherent states $|\alpha_{-1}\rangle$ and $|\alpha_{+1}\rangle$, which are eigenstates of $\hat{a}_{-1}$ and $\hat{a}_{+1}$, respectively. Then the state at later time t can be given by

$$\begin{aligned} |\psi(t)\rangle &= \exp(-iH_{\text{eff}}t)|\alpha_{-1}, \alpha_{+1}\rangle \\ &= \exp(-iH_{\text{eff}}t)|u_{-1}, u_{+1}). \end{aligned} \tag{24}$$

In order to simplify our the following analysis, we rewrite the Hamiltonian $H_{\text{eff}} = 2\lambda_2' \hat{A}_{-1}^\dagger \hat{A}_{-1} \hat{A}_{+1}^\dagger \hat{A}_{+1} + \Omega(\hat{A}_{-1}^\dagger \hat{A}_{-1} - \hat{A}_{+1}^\dagger \hat{A}_{+1})$ and use a scaled time $\tau = \lambda_2' t$ and a dimensionless parameter $K = \Omega/\lambda_2$, which leads the state to the simple form

$$\begin{aligned} |\psi(\tau)\rangle = e^{-1/2(|u_{-1}|^2+|u_{+1}|^2)} \sum_{n,m=0}^{\infty} \frac{1}{\sqrt{n!m!}} u_{-1}^n u_{+1}^m \\ \times e^{-i\theta_{n,m}\tau}|n, m), \end{aligned} \tag{25}$$

where we have used a running frequency $\theta_{n,m} = nm + K(n - m)$. Since our main interest is to create continuous-variable-type entangled states, we rewrite state (25) as the following integral form

$$\begin{aligned} |\psi(\tau)\rangle = \int_0^{2\pi} \frac{d\phi_{-1}}{2\pi} \int_0^{2\pi} \frac{d\phi_{+1}}{2\pi} f(\phi_{-1}, \phi_{+1}) \\ |u_{-1}e^{i\phi_{-1}}, u_{+1}e^{i\phi_{+1}}), \end{aligned} \tag{26}$$

where the phase function is given by

$$f(\phi_{-1}, \phi_{+1}) = e^{-i(\tau\theta_{n,m}+n\phi_{-1}+m\phi_{+1})}. \tag{27}$$

Equation (27) indicates that the state $|\psi(\tau)\rangle$ is a continuous superposition sate of two-mode product coherent states. From (25)–(27), we can see that the values of the K parameter, which is the relative strength and the coupling and the interaction between atoms, may affect the form of the state. Here our attention is paid to the situation where K may take nonzero integers values. It follows from (25) that $|\psi(\tau + 2\pi)\rangle = |\psi(\tau)\rangle$. This means that the time evolution of the state (25) is a periodic one with respect to the scaled time τ. On the other hand, suppose that the scaled time τ takes its value in the following manner

$$\tau = \frac{M}{N} 2\pi, \tag{28}$$

where M and N is mutually prime integers, a simple calculation gives

$$\exp\left(i2\pi\frac{M}{N}\theta_{n+N,m+N}\right) = \exp\left(i2\pi\frac{M}{N}\theta_{n,m}\right), \tag{29}$$

which means that the exponential phase function is a periodic function with respect to both n and m with the same period N. If τ takes its values according to (28), the state (25) can be decomposed into a discrete superposition state of coherent state

$$|\psi(\tau = \frac{M}{N}2\pi)\rangle = \sum_{r=1}^{N}\sum_{s=1}^{N} c_{rs}|u_{-1}e^{i\phi_{-1,r}}, u_{+1}e^{i\phi_{+1,s}}), \tag{30}$$

where the two running phase have the form

$$\phi_{-1,r} = \frac{2\pi}{N}r, \phi_{+1,s} = \frac{2\pi}{N}s(r, s = 1, 2, ..., N). \tag{31}$$

From Eqs. (25) and (30), we find the following equation to determine the coefficients c_{rs}

$$\sum_{r,s=1}^{N} c_{rs}\exp\left(-i\frac{2\pi}{N}(M\theta_{n,m} + nr + ms)\right) = 1. \tag{32}$$

Carrying out summations over n and m in left hand side of the above equation from 1 to N, and making using of the normalization condition $\sum_{r,s=1}^{N} c_{rs}c_{rs}^* = 1$, we arrive at the explicit form of the coefficients c_{rs}

$$c_{rs} = \frac{1}{N^2}\sum_{r,s=1}^{N}\exp\left(i\frac{2\pi}{N}(M\theta_{n,m} + nr + ms)\right). \tag{33}$$

It is straightforward to see that the discrete superposition state (30) is generally an entangled coherent state with N^2 independent product coherent states. As a specific example of creating the continuous-variable-type entangled state, in what follows we discuss the generation of the entangled state for the case of $K = 101$,$N = 2$ and $M = 1$. From Equation (33), we obtain the coefficients

$$c_{11} = c_{12} = c_{21} = -c_{22} = \frac{1}{2}, \tag{34}$$

which result in the following entangled state

$$|\psi\left(\tau = \frac{M}{N}2\pi\right)\right\rangle = \frac{1}{2}|u_{-1})|u_{+1}^{-}) + \frac{1}{2}|-u_{-1})|u_{+1}^{+}) \tag{35}$$

where $|u_{+1}^{\pm})$ are unnormalized atomic Schrödinger cat states, i.e., even and odd coherent quantum superposition states defined by $|u_{+1}^{\pm}) = |-u_{+1}) \pm |u_{+1})$. The degree of quantum entanglement of such an entangled state can be measured in terms of concurrence Wang (2001); Hill and Wootters (1997) (Wang, 2001; Hill and Wootters, 1997). The corresponding concurrence for our case is given by Kuang and Lan (2003) (Kuang and Zhou, 2003)

$$\mathcal{C} = \sqrt{[1 - \exp(-4|u_{-1}|^2)][1 - \exp(-4|u_{+1}|^2)]}. \tag{36}$$

If one turn to the Fock space of $(\hat{a}_{-1}, \hat{a}_{+1})$, he can find that the state becomes

$$\begin{aligned} \left|\psi\left(\tau = \frac{M}{N}2\pi\right)\right\rangle &= \frac{1}{2}(|\alpha_{+1}, \alpha_{-1}\rangle + |-\alpha_{+1}, -\alpha_{-1}\rangle) \\ &\quad + \frac{1}{2}(|-\alpha_{-1}, -\alpha_{+1}\rangle - |\alpha_{-1}, \alpha_{+1}\rangle), \end{aligned} \tag{37}$$

which is an superposition of even-like and odd-like coherent states.

4. CONCLUSION AND REMARKS

In summary, we have presented a theoretical treatment of the proposal for creating maximally entangled states of many particles in spin-1 Bose–Einstein condensates (BECs) by applying a single atom Raman transition. It is shown that in the strong coupling limit and certain initial states, the spin-1 system can be reduced to an effective two-component problem with the help of two simple dressed operator. We also give a scheme to realize the atom-atom continuous-variable entangled states in this system. Notice that an experimental result has been reported recently for localizing the numbers of atoms at an individual lattice site Greiner *et al.* (2002) (Greiner *et al.*, 2002). In the experiment, the average occupations per lattice site were around 1–3 atoms. As a simple example, we considered the interesting case Wu and Yang (2003) (Wu and Yang, 2003) of $N = 2$ and constructed the explicit entangled state. On the other hand, a scheme for demonstrating the entanglement swapping in trapped Bose–Einstein condensates is proposed Dunningham *et al.* (2002). Maybe, our method could find its applications and can be useful in further works.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No.10174007), the Natural Science Foundation of Hunan Province (Grant No.03JJY3012), and the Natural Science Foundation of Hunan University of Science and Engineering (Grant No. 05ZD005). The Human Education Foundation of the trained object for Human youth skeleton-teacher (05GGPYDX0907). One of the authors (X.L.X.) would like to thank Professor G. J. Yang, M. F. Fang, and Dr. Q. T. Xie.

REFERENCES

Barrett, M., Sauer, J., and Chapman, M. S. (2001). *Physical Review Letters* **87**, 010404.

Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., and Wootters, W. K. (1993). *Physical Review Letters* **70**, 1895.

Bennett C. H. and Wiesner, S. J. (1992). Physical Revieve Letters 69, 2881.

Cirac, J. I., Lewenstein, M., Mølmer, K., and Zoller, P. (1998). *Physical Review A: Mathematical and Journal* **57**, 1208.

Cleve, R. Gottesman, D., and Lo, H.-K. (1999). *Physical Review Letters* **83**, 648.

Duan, L.-M., Cirac, J. I., and Zoller, P. (2002). *Physical Review A: Mathematical and Journal* **65**, 033619.

Duan, L.-M., Sørensen, A., Cirac, J. I., and Zoller, P. (2000). *Physical Review A: Mathematical and Journal* **85**, 3991.

Dunningham, J. A., Bose, S., Henderson, L., Vedral, V., and Burnett, K. (2002). *Physical Review A: Mathematical and Journal* **65**, 064302.

Facchi, P. and Pascazio, S. (2002). *Physical Review Letters* **89**, 080401.

Gerry, C. C. and Campos, R. A. (2003). *Physical Review A: Mathematical and Journal* **68**, 025602.

Gordon D. and Savage, C.M. (1999). *Physical Review A: Mathematical and Journal* **59**, 4623.

Greiner, M., Mandel, O., Esslinger, T., Hänsch, T. W., and Bloch, I. (2002). *Nature (London)* **415**, 39.

Helmerson, K. and You, L. (2001). *Physical Review Letters* **87**, 170402.

Hill, S. and Wootters, W. K. (1997). *Physical Review Letters* **78**, 5022.

Hines, A. P., McKenzie, R. H., and Milburn, G. J. (2003). *Physical Review A: Mathematical and Journal* **67**, 013609.

Kuang, L.-M. and Ouyang, Z.-W. (2000). *Physical Review A: Mathematical and Journal* **61**, 023604.

Kuang, L.-M. and Lan Zhou, (2003). *Physical Review A: Mathematical and Journal* **68**, 043606.

Law, C. K. Pu, H., and Bigelow, N. P. (1998). *Physical Review Letters* **81**, 5257.

Loyd, S. (1993). Science **261**, 1589.

Micheli, A., Jaksch, D., Cirac, J. I., and Zoller, P. (2003). *Physical Review A: Mathematical and Journal* **67**, 013607.

Nielsen M. A. and Chuang, I. L. (2000). *Quantum Computation and Quantum Communication* (Cambridge University Press, Cambridge).

Pu, H. and Meystre, P. (2000). *Physical Review Letters* **85**, 3987.

Ruostekoski, J., Collett, M. J. Graham, R., and Walls, D. F. (1998). *Physical Review A: Mathematical and Journal* **57**, 511.

Search, C. P., Rojo, A. G., and Berman, P. R. (2001). *Physical Review A: Mathematical and Journal* **64**, 013615.

Simon, C. (2002). *Physical Review A: Mathematical and Journal* **66**, 052323.

Sørensen, A. S., Duan, L.-M., Cirac, J. I., and Zoller, P. (2001). *Nature (London)* **409**, 63.

Stenger, J. Inouye, S. Stamper-Kurn, D. M. Miesner, H.-J. Chikkatur, A. P., and Ketterle, W. (1998). *Nature (London)* **396**, 345.

Steel M. J. and Collett, M. J. (1998). *Physical Review A: Mathematical and Journal* **57**, 2920.

Wang, X. (2001). *Physical Review A: Mathematical and Journal* **64**, 022302.

Wu Y. and Yang, X. (2003). *Physical Review A: Mathematical and Journal* **68**, 013608.

Yi, S. Ö. Müstecaplioglu, E. Sun, C. P., and You, L. (2002). *Physical Review A: Mathematical and Journal* **66**, R011601.

You, L. (2003). *Physical Review Letters* **90**, 030402.

Zhang, M. and You, L. (2003). *Physical Review Letters* **91**, 230404.

Zou, X. Pahlke, K., and Mathis, W. (2004). *Physical Review A: Mathematical and Journal* **69**, 053608.